Enrollment No: ______________________ **Exam Seat No:** ____________________

C.U.SHAH UNIVERSITY

Summer Examination-2022

Subject Name: Complex Analysis

Subject Code: 4SC05COA1 **Branch: B.Sc. (Mathematics)**

Semester: 5 **Date: 22/04/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: **(14)**

a) Evaluate: $\int_c \frac{1}{z}\, dz$; C: $|z| = 1$. (02)

b) Is the function $f(z) = z^2$ is analytic? (01)

c) Define: Entire function (01)

d) A function $u(x, y)$ is said to be harmonic if and only if_____. (01)
(a) $u_{xx} + u_{yy} = 0$ (b) $u_{xx} - u_{yy} = 0$ (c) $u_{xy} + u_{yx} = 0$ (d) None

e) A function $f(z)$ is analytic if (01)
(a) Real part of $f(z)$ is analytic (b) imaginary part of $f(z)$ is analytic
(c) both (a) and (b) (d) None of these

f) If $f(z) = z - \bar{z}$ then $f(z)$ is _____. (02)
(a) Purely real (b) Purely imaginary (c) Zero (d) None

g) Which are the fixed points of $w = \dfrac{2z-3}{z+2}$? (02)

h) Define: Harmonic function. (02)

i) State C-R equation in polar co-ordinates. (02)

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions **(14)**

a) Show that $f(z) = \begin{cases} \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2} & ; z \neq 0 \\ 0 & ; z = 0 \end{cases}$ is continuous at origin. (05)

b) Suppose $f(z) = u + iv, z_0 = x_0 + iy_0$ and $w_0 = u + iv$ then $\lim_{z \to z_0} f(z) = w_0$ if and only if $\lim_{(x,y)\to(x_0,y_0)} u(x, y) = u_0$f and $\lim_{(x,y)\to(x_0,y_0)} v(x, y) = v_0$. (05)

c) Prove that $f(z) = \bar{z}$ is no-where differentiable. (04)

Q-3 Attempt all questions **(14)**

a) Show that $u(x,y) = 2x - x^3 + 3xy^2$ is harmonic. Find harmonic conjugate of $u(x,y)$. Also find analytic function. (05)

b) Evaluate $\int_C z^2\, dz$ where C is the path joining the points $z = 1 + i$ to $z = 2(1 + 2i)$ along the straight line joining $1 + i$ to $2(1 + 2i)$. (05)

c) Evaluate: $\int_c \frac{e^z}{(z-3)(z-1)}\, dz$, where c is circle $|z| = 4$. (04)

Q-4 Attempt all questions **(14)**

a) State and prove C-R equation in cartesian coordinates. (07)

b) Evaluate: $\int_C \frac{dz}{z^2+9}$ where $C: |z| = 5$. (05)

c) Find invariant points for $f(z) = \frac{3z-5}{z+1}$. (02)

Q-5 Attempt all questions **(14)**

a) Determine the analytic function whose real part is $e^{2x}(x \cos 2y - y \sin 2y)$. (05)

b) Find image of $|z - 3i| = 3$ under the mapping $w = \frac{1}{z}$. (05)

c) Transform the curve $x^2 - y^2 = 4$ under the mapping $w = z^2$. (04)

Q-6 Attempt all questions **(14)**

a) State and prove Cauchy's integral formula. (07)

b) State and prove ML- inequality. (05)

c) State Liouville's theorem. (02)

Q-7 Attempt all questions **(14)**

a) Evaluate: $\int_C \frac{z^3+z^2+z+1}{z(z-1)^2} dz,\ C: |z| \le 2$. (06)

b) State and prove Cauchy's theorem. (05)

c) Find arc length for the curve $c: z(t) = 1 - 3it, t \in [-1,1]$. (03)

Q-8 Attempt all questions **(14)**

a) Find the Mobius transformation that maps the points $z_1 = -1, z_2 = 0, z_3 = 1$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively. (07)

b) Prove that $\left|\int_c \frac{1}{z^2+1}\, dz\right| \le \frac{2\pi}{3}$, where c is the arc of the circle $|z| = 2$ that lies in first quadrant. (05)

c) If $u(x,y) = \frac{x(1+x)+y^2}{(1+x)^2+y^2}$, $v(x,y) = \frac{y}{(1+x)^2+y^2}$ then find $f(z)$ in terms of z. (02)